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THE INFLUENCE OF PROJECTION TRANSFORMATIONS ON DEGREE OF GENERALIZATION OF MAP CONTENTS

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Abstract: This is the study about geographic aspect of mathematical projections of the Earth's sphere surface on a flat surface (plane or map) and it's influence on degree of generalization. The cartographic generalization is creative process of generating, but it may be said that it is also one of factors that refers to presentation the geographic reality at a map. The geographic maps can get one new meaning, generated but connotative too, by generalization, because main characteristics of spatial occurrences are emphasis by graphical expression - map. Among these influence factors on process of generalization is also the type of transformation in projection that we use for constructing the map. This refers usual to small-scaled maps and to change of scale factor with changing longitude and latitude. In this paper, it will be presented with more details connection between generalization and cartographic projections which are used for presentation sphere ellipsoid to plane (plan or map). Scale factor is very important because with change of longitude and latitude it is also changing, and it is the main indicator of transformations. That is the reason why the scale factor is presented with more details in this paper.

Key words: generalization, projection, scale factor.

Абстракт: Овај рад обухвата географски аспект картографских пројекција сферне површине Земље на равну површ (план или карту) и њихов утицај на генерализацију садржаја карата. Картографска генерализација јесте стваралачки процес уопштавања који утиче на начин представљања географске стварности на карти. Генерализацијом карта добија се једно ново значење, уопштено али и конотативно, јер истиче главне карактеристике појава и процеса у простору преко графичког израза, карте. На степен и врсту генерализације утичу многи фактори. Међу факторима који утичу на степен генерализације географских елемената јесте и тип и врста деформација примењених картографских пројекција. Ово се углавном односи на ситноразмерне карте где је очигледна промена делимичног размера са променом географске дужине и ширине. У овом раду биће детаљније представљена међузависност генерализације и деформација картографских пројекција којим се сферни елипсоид пројектује на раван (план или карту). Са променом географске ширине и дужине мења се и делимични размер који је уједно и главни показатељ деформација због чега ће се у овом раду значајније бавити овом проблематиком.

Кључне речи: генерализација, пројекција, делимичан размер

The Introduction

The cartography generalization is one creative process of generating, and refers to creating and designing the contents of geographic maps. The main goal of generalization process is to establish the regular development of spatial occurrences for mapping. Beside the basic components of cartography generalization like good overview, legibility, visualization, in this process, i.e. good quality of the map, it is also used cartometry and interpretation and all together contribute to determination of regular development of spatial phenomenon.

Methods of Geography Generalization

Method of cartography generalization is “wide area for scientific researching in cartography…but there isn’t any solution or recipe for solution…the solution is different for each map individually” (Milisavljević, 1974). The cartography generalization is based on scientific analysis and synthesis of geography reality and its’ parts and also it is based on logical-methodological acts of abstraction and generating the mapped area. The process of cartography generalization implements in several ways:

1. “selection, i.e. reduction of geographic data,
2. simplification,
3. symbolization,
4. exaggeration,
5. classification,
6. induction,” (Robinson A. et al.,1995)
7. “contracting (compressing) qualitative and quantitative characteristics,
8. transformation of occurrences’ groups into higher level concept and
9. unification of homogeneous occurrences.” (Milisavljević, 1974)

There are also several methods that we can emphasize in process of generalization:

1. “method of chosen contents,
2. method of simplifying shapes,
3. method of resume,
4. method of generalization by qualitative change and
5. method of pushing back the contents.” (Lješević, Živković,2001)

Special type of influence on degree of any type of generalization at some parts on maps, is the type of map projection. This influence refers to change of scale factor with growing latitude or longitude which depends on type of projection (polar, equatorial or horizontal). Maps with same scale, purpose and same territory of mapping, constructed in different projections, have also different degree of generalization at some parts of the maps. The way of understanding this distinction is in explanation of transformation that appears with changing longitude and latitude and scale factor with it.

The Factors of Generalization

The geographic (thematic) contents at territory we are mapping, have to be in main scale so it can't be overloaded which is realized by special generalization. This difference in degree of generalization is very sensible and depends on territory that is mapped. The factors of this type of generalization are:
- "purpose of mapping,
- main map scale,
- territory for mapping,
- sources (origins) for mapping,
- stage of readability,
- optimal graphical weight" and (Milisavljević S.,1974)
- type of projection.

Different type of map projections at example of one same territory have different scale factors at different latitudes which depends of type of projection. This is one special influence to degree of map generalization.

The Scale Factor and Tissot`s Indicatrix

The map or plan scale is relation of any length at map or plan to it's horizontal projection (in nature) on a ground.
In mathematical cartography:

$$c = \lim \frac{A'C'}{AC}$$

$$AC \to 0$$

$$c = \frac{dS}{ds} \vee \frac{dP}{dp} = 1$$ (Borčić 1955)

c – scale factor
dS or dP – infinitely small length or area in projection
ds or dp – infinitely small length or area at ellipsoid
This relation with projection which keeps equality of length or area looks like this

$$c = \frac{dS}{ds} \vee \frac{dP}{dp} = 1$$ (Borčić 1955)

The surface of ellipsoid can not be spread into plane without clefts and wrinkles. This is the reason for changing the scale from point to point and at one point i.e. around one point at different directions. The scale that we used for decreasing ellipsoid is main scale and it is usual marked at maps. The projection at plane at different places has different scale. These different scales we call scale factors (SF). The precision of projection is determinate, beside other factors also by difference between main scale and scale factor. If main scale is equal 1, than difference between scale factor and 1 we call transformation or distortion.

$d_c = c - 1$

d_c - transformation of length or area

c - scale factor (SF)

Scale factor depends on point's position and also of linear element's azimuth α. Transformation of angles we can consider as transformation of angles' sides that they make with main directions.

$d_u = U - U'$

d_u - angle transformation

U - angle at ellipsoid

U' - angle in projection (Borčić 1955)

Scale factor has the largest values for azimuth values α and α + 90°. For one of these values, scale factor will be the largest and for another will be the least. In every point at ellipsoid (sphere) there are two orthogonal directions which also in projection remain orthogonal. Along these directions scale factor has the largest and the least value. These directions we call main directions.(Borčić, 1955)

Tissot used graphic device, the indicatrix, to illustrate the angular and area distortions that occur at points as a result of transformation. At any point on the globe, the scale factor is the same in every direction. Therefore, scale factor (SF),

SF = a = b =1. (Robinson et al. 1995)

We have already represented a point by infinitely small circle with a radius of 1.0. This basic system of transformation in projections is demonstrated by magnitudes of a and b which ordinary would be different than 1.0. When a is not equal to *b*, the indicatrix circle is transformed into an ellipse. As an example (Fig.1.) is shown the illustration of indicatrix of transformations. (Arnheim, R., 1976)

On a globe, SF in every direction is 1.0. so circle constructed with OA =1= radius r. In the transformations, the directions OA and OB are principal directions. Those are the pair of orthogonal directions on the globe that are retained as orthogonal by the transformation.

The circle of the indicatrix in equal-area projections are deformed like:

OA'= *a* = 1.25 and OB'= *b* = 0.80 (the product)
The circle the indicatrix in conformal are deformed like:
OA'= *a* = 1.25 and OB''= *b* = 1.25 (the product).
The area is enlarged by factor of 1.5625, but *a = b.*
The angle MOA=*U* on the globe becomes M'OA'= *U'* in projection. *U*-*U'*= Ω and it denotes maximum possible disorder that may occur (2Ω).(Robinson A. et al.,1995)

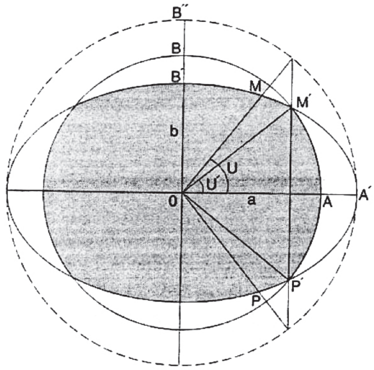

Figure 1. The indicatrix of transformations. (Robinson A. et al.,1995)

Types of Geography Projections by Character of Transformations

Cartographic projections are divided into conformal, equidistant, equal-area and arbitrary projections by the type of transformations, i.e. by keeping mathematical values like angle, length, area or transformations of all these elements. This division is very important because transformation of these elements has influence to generalization of areas, lines and angles at large latitudes of the map.

Conformal projections are constructed by keeping unchanged values of the angles. Angles between several directions on the map are equal as ones between the same directions on the ground.

m - module of scale factor by meridians

n - module of scale factor by parallels

In conformal projections $m = n$

$a=b=c=m=n; \alpha=A$

α – angle in projection

A – angle on Globe

Conformal projections are: *Mercator Cylindrical, Lambert Conic, Azimuthal stereographic, etc.* (Maling, D.H, 1992)

Equal-area projections are constructed by keeping unchanged values of the areas. The condition of equal-areas is:

$p = mn - 1; \; p = ab = mn\sin\Theta = 1$

Θ – angle between meridians and parallels in projection

In these projections infinitely small circle is deformed into ellipse with equal values of areas and also, every figure of arbitrary area is deformed into areas` equivalent figure.

Equal-area projections are: *Lambert Cylindrical, Conic, Azimuthal, Goode Homolosine, Ekcert thorus, Sanson Pseudocylindric,* etc.

Equidistant projections are projections that have scale factor stated as 1 by one of the main directions. In these projections angles and areas are deformed. Some of these projections are: *Ptolomey Conic, Square or Rectangle Cylindrical, Postel Azimuthal, orthographic projections,* etc.

$a = 1$, or $b = 1$ (Maling, D.H, 1992) $\mu = \frac{r'}{r}$

Types of Cartography Projections by the Constructing Area

Beside this classification, the projections can be divided by type of surface for constructing (projecting) the net of parallels and meridians from the Earth to that surface. They can be Azimuthal, Cylindrical or Conic. The Azimuthal projections are constructed by projecting the meridians and parallels from the Earth surface to the plane. The Cylindrical projections are constructed by projecting the meridians and parallels from the Earth surface to the cover of cylinder. The Conic projections are constructed by projecting the meridians and parallels from the Earth surface to the cover of conus. There are also subgroups of these projections like: Polyconic, Pseudoconic or Pseudocylindar and Polyhedron projections.

Types of Azimuthal Projections by Center of Projection

The Azimuthal projections can be divided also by the method of projecting net of parallels and meridians at a plane (map). The plane is usual concerned as touches the Earth (or model – globe) in some point. There are: orthographic, stereographic and central projections.

In orthographic projections, the center of projection is imagined at infinitely distance from the Earth. The projection plane is parallel with projection rays and it can be at different distance from the Earth and this factor does not have influence on the principle of projection.

In stereographic projections, the center of projection is imagined at surface of the Earth but at the different side from projection plane. In central projections, the center of projection is imagined at the center of the Earth.

The scale factor can be larger than 1 or smaller than main scale (1) which depends on type of projection and its' transformations along meridians and parallels. Using this rule, there will be presented different influences of changing scale factor along meridians and parallels on degree of generalization at special areas on maps.

The Deformation of Azimuthal Projections

The conclusion of the study of these projections is that the scale factor is changing from 1 to 0 in orthographic projections (polar and equatorial) along the meridians and they are equidistant along parallels. In stereographic projections scale factor is changing from 1 to 2 from center to circumference of the circle. They are conformal projections. In central projections scale factor is changing from 1 to endless along the meridians. If we compare Azimuthally orthographic polar equidistant projections and Azimuthally stereographic conformal projections, the changing of scale factor differs in the way that decreases from 1 to 0 (from pole to equator) in orthographic projection, and increases from 1 or ½ to 2 at equator in stereographic projections. From these facts we can conclude that these two types of projections are different in manner of generalization special with changing the latitude or longitude (this depends if the projection is equatorial, polar or horizontal). In equatorial projections, this changing of scale factor is along the parallels and in polar, along the meridians. Azimuthally orthographic projections have higher level of generalization, specially at equator's area, then azimuthally stereographic projections. (Snyder, J. P. 1985)

Pastel's projection is equidistant. Scale factor is keeping it's value (1) along the meridians and it is the same as main scale for the map. Along the parallels, the scale factor is growing from center to map boundary. The value of the scale factor is even higher than 1.5 at equator area and this is the area of smaller degree of generalization.

The Deformation of Cylindrical Projections

The cylindrical projections can contain constructed parallels at same distance (equidistant) or at different distance (conformal). Equidistance of the meridians is saved. The touching and cutting parallels are by rule without the transformations, but with a increasing the distance from them, the transformations are larger. We can discuss about the transformations that appear with increasing the latitude of Squared, Rectangle and Mercator`s projections and also about degree of the generalization at maps constructed by these projections.

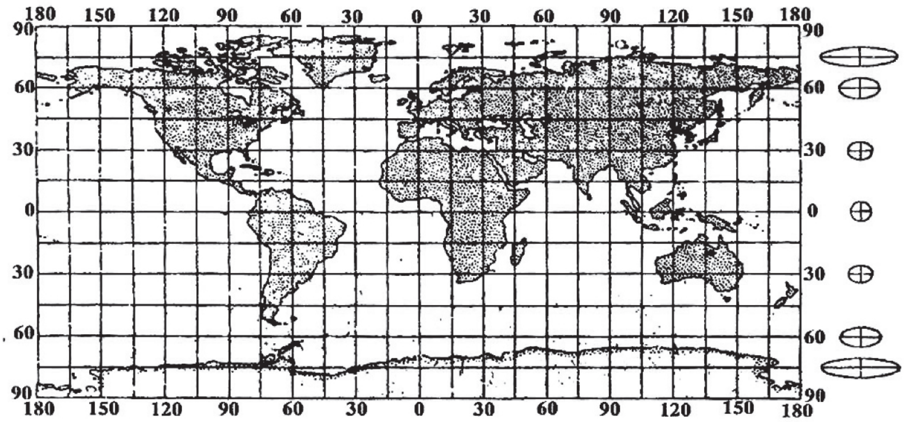

Figure 2. Meridians and parallels in Squared projection and the indicatrix of transformations. (Lješević, Živković,2001)

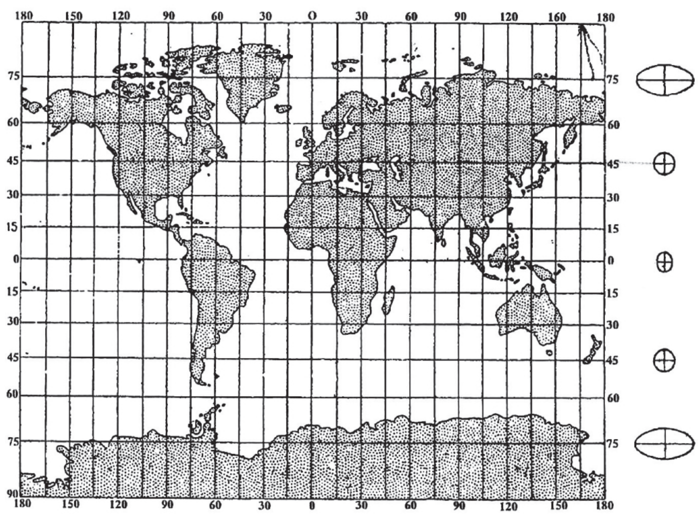

Figure 3. The Rectangle projection with indicatrix of transformations. (Lješević, Živković,2001)

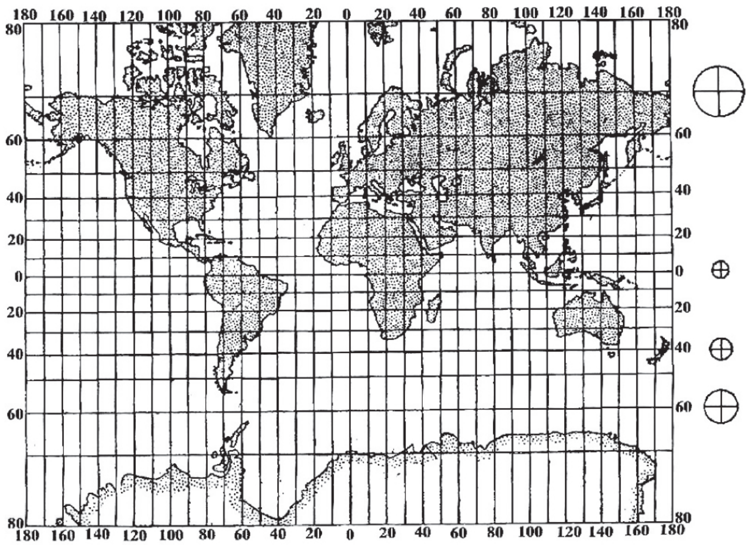

Figure 4. Mercator's projection with indicatrix of transformations.
(Lješević, Živković,2001)

In Squared projections scale factor at parallels is increasing from equator to poles. At equator it's value is 1 and at poles is endless. The scale along meridians is unchanged and the projection is equidistant along meridians. Transformations of angles around equator are insignificant but at larger distance from it, they are bigger. (Buttenfield, B.P., and R.B. McMaster, 1983.)

The Rectangle projections are equidistant. Scale factor is same as main scale along the cutting meridians (i.e. 45° North and South latitude). Between them, the areas are deformed by North-South direction (scale factor is larger than 1), and at latitudes higher than 45° scale factor is larger than 1 along the parallels, so the areas are deformed by East-West direction.

Mercator's projection is conformal. Angles have same value but areas and distances are deformed. That is the reason for using them in making sea-maps for defining loxodrome (the rhumb line which has constant azimuth and presents a line that intersects each meridian at same angle) and orthodroma (true direction which presents the shortest route between two points on the globe). The distances between parallels are increasing from equator to poles. The poles can't be presented at this map because they are projected into endless. Scale factor is increasing from equator to poles, and the transformations along the meridians and parallels are bigger with increasing latitude.

Table 1. Parallel review of influence of changing scale factor, type of projection and type of transformation on degree of generalization

Type of projection	Type of transformation	Value of scale factor (c)	Degree of generalization
Azimuthal			
orthographic			
polar	Equidistant along parallels	Pole equator $\mathbf{1} \geq c \geq 0$ *along meridians.*	Depends of main scale and territory for mapping. It can be: -height -medium and -small.
equatorial	Equidistant along parallels	Equator pole $\mathbf{1} \geq c \geq 0$ *along meridians.*	The class limits such as 0-2, 2-4, 4-8, and 8-16. are called range-grading.
horizontal	Arbitrary projection (transformations of areas, lengths and angles)	center end $1 \geq c \geq 0$ *along parallels,* $c = 1$ *along meridians.*	The change from 2 to 4 is the same relative as change from 50-100% of contents of specific area. (Robinson A. et al.,1995)
stereographic			
polar	Conformal	Projection area touching the Earth: From center $½ \leq c \leq 1$ Center end $1 \leq c \leq \infty$ Projection area at surface of big circle: Center end $1 \leq c \leq 2$ *along meridians and parallels.*	Collapsing: line to point, area to point, area to line. Aggregation of areas and volumes. Elimination of points, lines and areas. Smoothing features (smoothing operations and surface-fitting models).

equatorial	Conformal	Projection area touching the Earth: From center $\frac{1}{2} \leq c \leq 1$ Center end $1 \leq c \leq \infty$ *along meridians and parallels* Projection area at surface of big circle: Center end $1 \leq c \leq 2$	Simplification (to determine important characteristics of feature attributes and eliminate unwonted detail). Classification (to express the main character of a distribution i.e. Ordering, scaling and grouping)
horizontal	Conformal	Center end $1 \leq c \leq 2$ *along meridians and parallels.*	Exaggeration (to enhance or emphasize important characteristics of the attributes).
Central			
polar	Arbitrary projection	pole equator $1 \leq c \leq \infty$	Symbolization (translation features to graphic marks-symbols on map)
equatorial	Arbitrary projection	equator pole $1 \leq c \leq \infty$	
horizontal	Arbitrary projection	Center end $1 \leq c \leq \infty$ *along meridians and parallels.*	
Cylindrical			
Squared	Transformations of angles and areas	Equator poles $1 \leq c \leq \infty$ *along parallels,* $c = 1$ *along meridians.*	
Rectangle	Transformations of angles and areas, Equidistant	$c = 1$ *Along cutting parallels,* $c = 1$ *Along cutting meridians.*	

Mercator's	Conformal	Touching cylinder: $c = 1$ *along parallels,* Center end $1 \leq c \leq \infty$ *along meridians.* Cutting cylinder: Between cutting parallels: $c \leq 1$ from cutting parallels to poles: $c \geq 1$ *along meridians.*	
Lambert's	Equivalent (transformation of angles)	Equator pole $1 \leq c \leq \infty$ *along meridians,* $c \geq 1$ *along parallels.*	
Gaul's	Arbitrary	Equator cutting p. $0 \leq c \leq \infty$ Cutting p. end $1 \leq c \leq \infty$ *Along meridians,* $c = 1$ *Along cutting parallels.*	
Conic			
Straight	Conformal Equidistant	$c \neq 1$ *along meridians,* $c = 1$ *along parallels.*	
Transversal		$c = 1$ *along meridians* $c \neq 1$ *along parallels.*	
Pseudo cylindrical			
Sanson's	Equivalent	$c = 1$ *along central meridian and parallels* $c \geq 1$ *along meridians.*	

*Malady'***s**	Equivalent along meridians	*c* = 1 *along meridians and equator* *c* ≤ 1 *along parallels*	
*Eckert***'***s*	Equivalent	*c* = 1 along meridians and equator line of pole = ½ equator	
Pseudo conic			
*Bon***'***s*	Arbitrary	*c* ≠ 1 *along meridians and parallels*	
Arbitrary			
*Grinten***'***s*	Between Conformal and Equivalent	*c* ≥ 1 *along meridians* $\varphi \geq 60^{O}$	
Plyconic			
*American Haler'***s**	Arbitrary Equidistant	*c* = 1 *along parallels and main meridian* *c* ≥ 1 *along meridians*	
Combinated projections			
*Auto'***s**	Equivalent	*c* ≥ 1 *along meridians and parallels*	
*Cut by Good***'***s method*	Equivalent along meridians constructed like six Molvayds' projections	*c* = 1 *along meridians and equator* *(areas along meridians* $\varphi \geq 60^{O}$ *c* ≤ 1 *along parallels*	

The consequences of these transformations at large latitudes is that we have maps of the same territory (i.e. World Map), constructed in different cylindrical projections (Square, Rectangle or Mercator) with different contents i.e. different degree of the generalization. The contents of larger latitudes at Mercator's projection are more complete than the contents of same area at Square projection.

The Deformation of Conic Projections
The Conic projections are constructed by conditionally projecting the coordinate net at imagined area of conic wrapper. They are used for presenting some parts of Earth surface or countries.

Straight conic projections have scale which is changing by meridians and along the parallels is unchanged. These projections are suitable for presenting the areas (countries) that are deformed by longitude. Transversal conic projections have constant scale by meridians as one by touching parallel, but along the other parallels is growing with dismissal from touching parallel. (Yang, Q., et al., 2000)

The Conclusion

Maps of the World, Continents or their parts are the small-scaled maps with different scale on their areas. They are the object of this study. Small areas` maps are usual constructed without this factor of generalization (map projection). Usual the large-scaled maps are maps of states, their parts or regions. They choose projection with less deformations at their parts because they are used often for measurement of distances, angles, azimuth etc. These projections are used for constructing topographic maps but also the thematic maps. The difference between small-scaled and large-scaled maps are beside the size of territory, the contents of map which depends on map thematic and purpose of map.

Transformations of the areas and angles are the condition of different degree of generalization in the maps of same territory. The contents of areas can be reduced if the scale factor at those latitudes or longitudes is smaller than 1. These facts are opposite with scale factor larger than 1. The conclusion is that equal- area projections are with same geographic contents because the equality of areas is kept. Conformal projections have transformations of the areas value but the similarity of the areas' shapes with areas on the globe are kept. These are reasons for different degree of generalization of contents of the same territory constructed by different projections on plane (map).Usual, in practice, the projection that is used for mapping of specific area is chosen by criterion of different type of transformation. That criterion depends of many factors. Frequently, when we have map in one projection, it is necessary for some needs to change the projection of the same map. That is the moment when generalization of the contents of the map is actual by using those 9 implement factors (methods of generalization). The generalization usual isn't radical but it can change attitude about space relations at a ground, which can have influence in making decisions for space governing. The generalization of these elements (lines, areas, angles) can't be without human activity, automatized by some kind of algorithm, because the degree of deformations differs from part to part of the map. The simplification is usual the main method of generalization beside selection, exaggeration, symbolization and classification. The simplification of lines, areas and different shapes are very often connected with reduction of elements which is important for constructing the map and it's graphical weight. Beside physical-geographic maps, very important element for analysis are thematic small-scaled maps. The symbol-scale problem is here independent factor which is loading map with contents of different shape and values, but that is not the objective of this paper.

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