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MATHEMATICAL-GEOGRAPHICAL ANALYSIS OF ORIENTATION OF CHURCH OF THE HOLY CROSS IN NIN

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Abstract: The paper contains mathematical-geographical and topographical analysis of the medieval XI century church of the Holy Cross in the city of Nin (Croatia). The motive for performing the analysis was a widely accepted opinion, which was based on the statements presented in the book "Number of the Light" (Pejaković, 1978) - that the orientation of the church has a solar meaning, or, more precisely, that the obvious irregularities in the construction of the church were a result of the master builder's intention to create a building which would simultaneously serve as a house of prayer, a calendar and a sundial. However, the results of this analysis do not prove this tempting story: the orientation of the church of the Holy Cross is dictated by the orientation of the surrounding earlier buildings, in the midst of which the church was constructed, while the distortions of the ground plan are a consequence of the medieval master builder's lack of education (and not of his intention to adapt the ground plan to the geometry of the Sun). Although it is a misperception, the theory on the solar meaning of orientation of the church of the Holy Cross is not completely without meaning: for the first time on the territory of ex Yugoslavia, this theory drew attention to the importance of investigating the meaning of orientation of medieval churches.

Key words: Church of the Holy Cross, mathematical geography, orientation, Middle Ages

Introduction: Church of the Holy Cross in Nin

In the Dalmatian island town of Nin (Croatia), historical seat of Croatian princes, there is a small medieval pre-Romanic church of the Holy Cross, which is figuratively called "the world's smallest cathedral" (Figure 1). Although more recent Croatian sources mention IX century, church probably built in the XI century (Vežić, 2013). The Holy Cross belongs to the central church type with a base in the shape of an inner Greek cross, central dome on a high circular-like drum and a spindle-shaped steeple which was constructed additionally; the central apse is surrounded by rectangular walls. The side apses which are joined with the central apse are topped by semi-calotte arches (Figure 2a). The church had not been painted. Its length is 9 m, which is approximately equal to the length of the span between the side arms. The ground plan is distorted, with no

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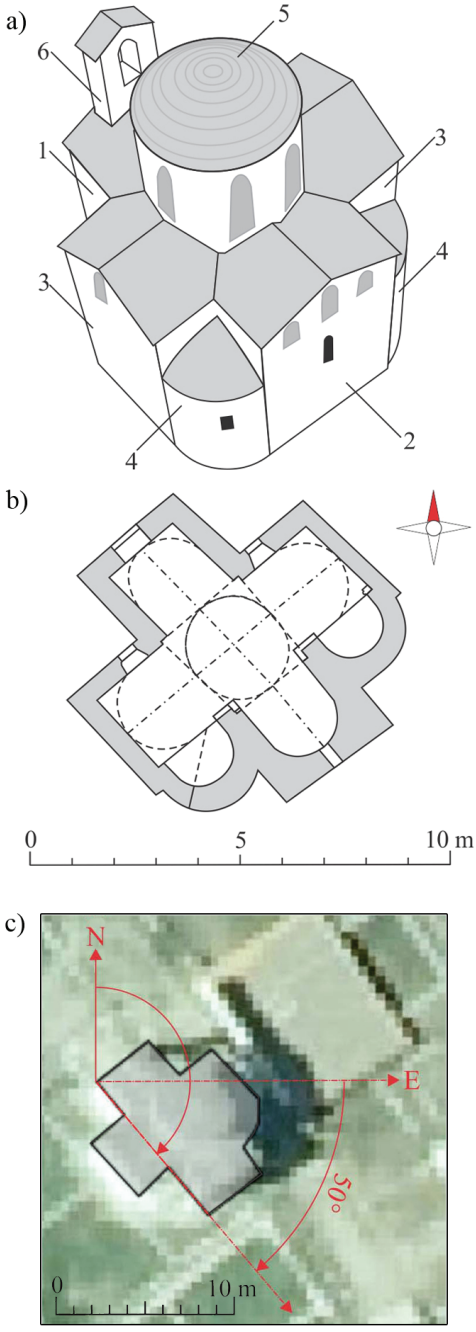
right angles, and without the main axis which would be the line of symmetry of the entrance and the frontal chapel (Figure 2.b).

Figure 1. a) the orthophotograph of Nin (the little cross represents the church); b) the orthophotograph of the St. Cross church (http://geoportal.dgu.hr/); the photographs of the St. Cross church: b) the view from the SW (door-side); d), the view from the SE (apse-side); e) the view from the SW; f) and the view from the S.



S.Church of the Holy Cross, as well as the church of St. Donates in Zadar, is one of the most famous old Croatian churches, mostly owing to painter and art historian M. Pejaković (1928–2005), which argued that irregularities of construction were actually "regular irregularities", rather than a consequence of master builder's lack of education (Pejaković, 1978; 1981; 1982). According to M. Pejaković, old Croatian master builder intentionally deformed the ground plan and designed an asymmetrical arrangement of apses and windows in order to create a complex building which would at the same time serve as a house of prayer, a reduced calendar (outer structure) and a religious sundial (inner structure).

Figure 2.

a) the diagram of the St. Cross church: 1) the door branch; 2) the front branch with the semicircular apse framed by the rectangular walls; 3) side branches; 4) the apses of the side branches; 5) the central circle-like dome on a high drum; 6) the distaff-shaped belfry built afterward;

b) the ground plain of the St. Cross church (Eitelberger R., 1861), orientated by the orthophotograph (c);

c) the simplified contour view of the St. Cross church at the orthophotomap (http://geoportal.dgu.hr/).

Not everybody has accepted this attractive story, but, by some miracle, there were no serious critics of this theory:

it is possible that historians of architecture and geographers have submitted to unclear and incomprehensible explanations of geometrical constructions of Pejaković's book, while the astronomers had little time to engage in earthly topics, or were all these experts just hesitant to question statements that glorify old Croatian masonry? Result: having taken the story for granted, the curious audiences have since 2009 started gathering around the church of the Holy Cross at the time of the summer solstice, in order to meet the sunrise, just like the New Age druids at Stonehenge: "The First Festival of Sun and Light at the church of the Holy Cross in Nin has taken place during the first day of summer in this royal city, starting a new tradition of an event that is new and unique for our region. The programme started early in the morning, at the time of the solstice itself, when the gathered had the opportunity to observe the play of sunlight created by a special arrangement and shape of the windows" (Stagličić, 2009).

So what was missing was an exact analysis, like those that have been carried out over the last couple of years on churches in different Balkan countries (Illiades, 2006; Antonaki, 2007; Tancheva, 2010; Tadić, 2012; Shumka, 2013 and others), an analysis like the one we shall carry out in the following chapter.

Analysis of the Orientation of the Church of the Holy Cross: Results and Discussion

In continuation, in an ordered manner (Articles 1–12), we have quoted main statements from the above mentioned book by M. Pejaković (1978) (except for the "non-astronomic" statements in connection to the church holidays). For each of them, we have quoted, in brackets, numbers of corresponding pages and illustrations in the book. Each statement has been commented on, with the difference that the numbers in the brackets correspond to the illustrations from this article.

1. The main axis of the church is not positioned along the west-east direction, the main apse is positioned towards the Southeast (pg. 31, Figure 3).
Comment. This is not directly stated in the book, but becomes obvious from the drawings contained in the book. Also, the book does not give us the deviation angle of the main axis of the church from the eastern direction, which is approximately $\Delta A = 50°$ (Figure 2.c). "Approximately", because the church axis follows a broken line (entrance side chapel - area under the dome-main apse); $\Delta A = 50°$ refers to the axis area under the dome/trapezoid).

2. The ground plan of the church fits into a square (pg. 29, Figure 2).

Comment. This is correct, and also normal: the planned ground plan was in the shape of an even-sided Greek cross.

3. The diagonals of this described square deviate from the cardinal directions by the angle α (pg. 31, Figure 3), which is a small angle but "of such great significance" (pg. 32). The walls of the side chapels also deviate from the sides of the described square also by the value of the same angle (pg. 31, Figure 3).

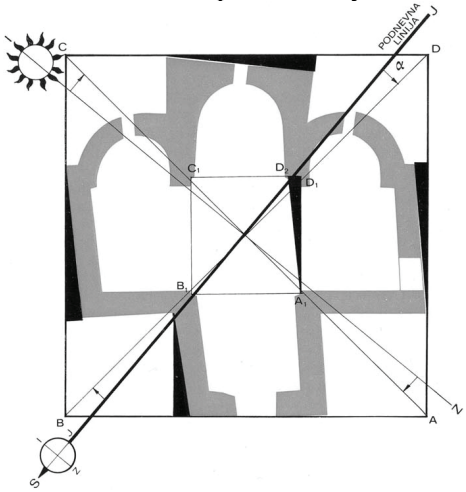


Figure 3. Ground plan of the church of the Holy Cross, described the square (ABCD), under the dome ($A_1B_1C_1D_2$), deviation angle (tinted triangles) and a hypothetical line of sunrise at the time of equinox (AC) (Pejaković, 1978, pg. 29).

Comment. Deviation angle α is the key point of the whole story, and has many times been mentioned in the book, but, interestingly, not once has the value of this angle been given; when measured on the ground plan, $\alpha = 5°$ (Figure 3) (see the final sentence of Article 5). Indeed, the deviation angles between the arms of the church and the sides of the described square are not the same (are not equal to angle α), but vary within the range of 3–8°.

4. A diagonal of the described square is defined by the direction of the first ray of rising sun at the time of the equinox, i.e., is directed to the point of the "true East" on mount Velebit (pg. 33, Figure 4).

Comment. What can be concluded from this is that the master builder marked upon the ground the direction towards the point of the sunrise above the visible horizon during the equinox, probably in spring (taking into account the duration of the construction season). He then drew the diagonal of the square (into which he later fitted the ground plan of the future church) along this direction. It is well known that this is one of the ways in which the medieval master builders defined the longitudinal axis of a future church, but the diagonal... This is something completely new?

5. The point of the rising sun of the vernal equinox on the visible horizon of the church of the Holy Cross deviates from the point of East to SE, for the value of angle α (pg. 35, Figure5).

Comment. Accordingly, the morning amplitude of the 21st March is a = α =5°. After a natural topographic profile is constructed on corresponding sheets along the continuation of the mentioned diagonal, it is possible to define the amplitude of the Sun at the moment of sunrise in this direction (h=0.8°), and then the value of the declination of the Sun (δ = –3,02°) and the corresponding vernal date of the modern calendar (12th March), and the Julian calendar in the XI century (6th March) (Tadić, Petrović, 2011). The master builder of the church of the Holy Cross, then, could not determine this direction in the XI century by observing the rising Sun on the 21st March (Figure 4, Figure 5). (Indeed, in XI century the vernal equinox, according to the Julian calendar, took place on 15th March, but the builders of that time were unaware of this fact).

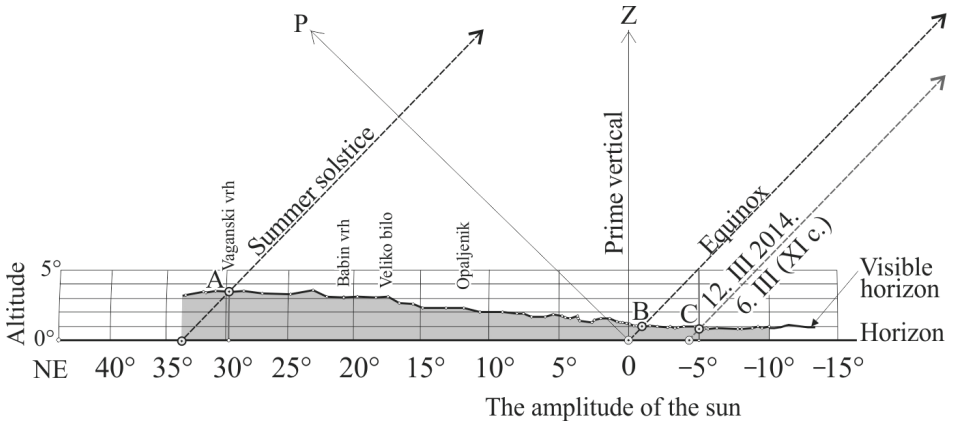


Figure 4. Geometrized sector of the visible horizon of the church of the Holy Cross in the transverse orthographic projection, in the azimuth range from $A_N = 55°$ to $A_N = 95°$; (A, B, C) - the point of actual Sun rising during the summer solstice, both equinoxes and on 12th of March (the azimuth of sunrise $A_N = 95°$).

The explanation of the direction of the diagonal is actually completely simple: by adapting to the existing urban environment (which is a very common practice), the master builder of the church of the Holy Cross has turned the described square of the ground plan by 50° relative to the cardinal points, so that the diagonals make angles of 5° relative to the cardinal directions, and there is really nothing unusual about this.

6. At the sunrise, during the equinox, the shadow of the northwest wall of the left arm of the church exactly covers the northeast wall of the entering arm (page 41, figure 9). The master builder achieved that calendar effect by giving the same bevel to the both walls, northwest and northeast.

Comment. But the situation mentioned above actually doesn't happen on March 21st and can not be an accurate announcement of the equinox. However, why should master builder, being educated in astronomy, curved the walls in order to capture equinox (figure 6), if he could find simpler solution, for example, to put northeast window of the left arm of the church in purposeful position for equinox. The other simple solution would be to put the window on the wall of the apse in the right position to capture equinox.

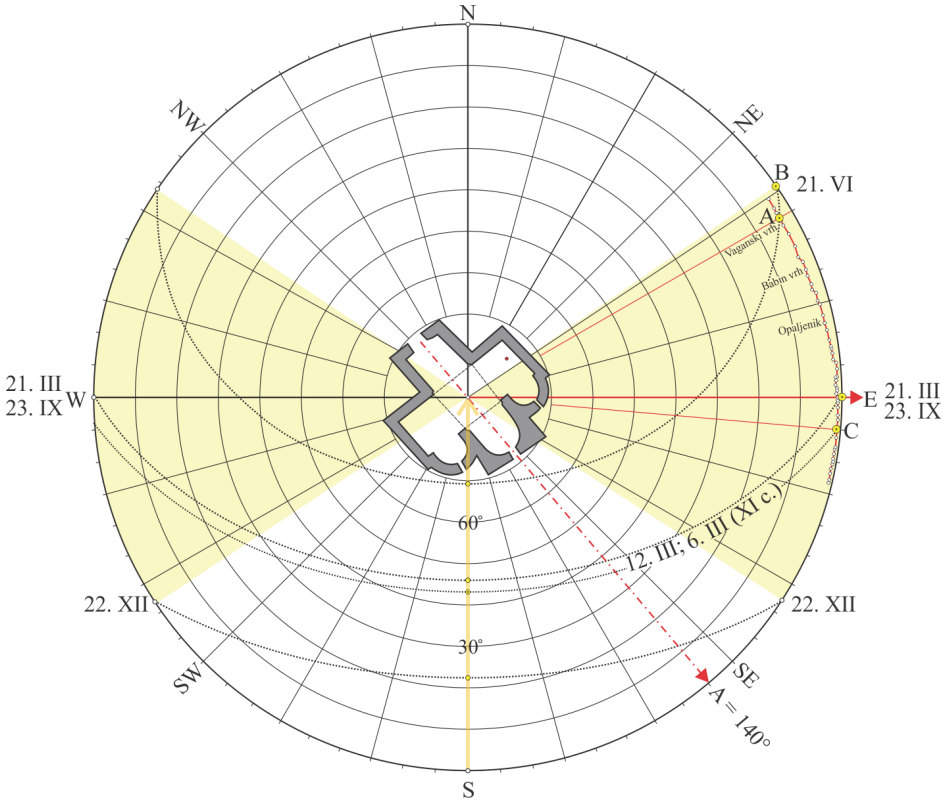


Figure 5. The upper celestial hemisphere shown in the azimuthal equidistant projection, with the Holy Cross Church as the constructional pole (44° 14' 34" N, 15° 11' 04" E), with part of the visible horizon (see Figure 4), alt-azimuth coordinate grid (Δh = 10°, ΔA = 15°) and diurnal arcs of apparent paths of the Sun for the solstice and equinoxes. In the middle, with the center of the under the dome, is the set-oriented ground plan of the Holy Cross Church. The morning (eastern) and evening (western) sectors of horizon which are bounded by solstice sunrises/sunsets, the total range of 67° 28' (that is, 33° 44' times two) are emphasised on the cartographic grid. The apparent path of the Sun is shown on the grid on 12th of March, when the Sun rises over the visible horizon (over Velebit Mountain) at point S, which is 5° offset from the E to SE.

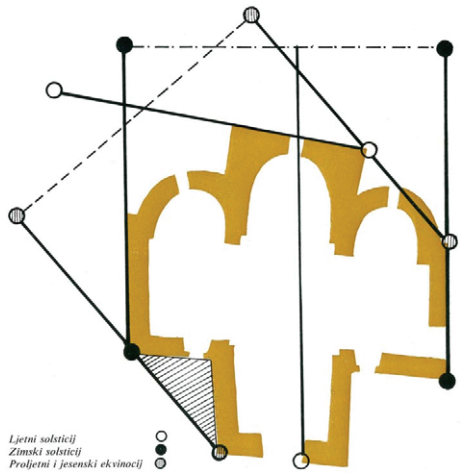


Figure 6. (Pejaković, 1978, pg. 79)

7. The directions of the walls of the side arms of the church are defined by the first rays of the rising sun of the winter solstice, "dislocated for angle α" (pg. 41, Figure 9).

Comment. Dislocated for angle α, in which direction, and why? If, e.g., the equinox point of the actual sunrise is dislocated for angle α towards SE, this doesn't mean that the real solstice points of sunrise are automatically dislocated for the same angle, this depends on the landscape (Figure 5, Figure 6). Besides, this statement is not correct, the side walls do not have an azimuth of the actual sunrise during the winter solstice, whether it be plus or minus angle α, which is 5° (Figure 6, Figure 7, left). And would this mean that the master builder was so patient/idle as to draw/distort the ground plan of the future church from vernal equinox to winter solstice?

8. The frontal wall of the main apse is directed "in the direction of the east of the summer solstice", i.e., to the point of the sunrise of the summer solstice (pg. 40–41, Figure 9).

Comment. If we take a continuous observation of the actual points of sunrise on the visible horizon (instead of switching our attention from points on the visible horizon to the theoretical points on the mathematical horizon), this will not prove correct: azimuth of the actual point of sunrise, dislocated towards the eastern point because of the landscape forms (the ridge of mount Velebit), does not equal the azimuth of the frontal apse wall (Figure 5, Figure 7, left).

9. The side apses are dislocated so that the angles between the lines drawn from the point on the entrance doorstep to the windows of the apses and the longitudinal axis of the church would be 23,5° (pg. 47, Figure 14), and the angles between the lines drawn from the centre of the base of the area under the dome to those same windows would be equal to the solstice morning and evening amplitudes of the sun on the horizon of the church of the Holy Cross (pg. 48, Figure 15).

Comment. The master builder has, then, distorted the side apses in order to permanently "record" within the church structure the magnitude of the angle of obliquity to the ecliptic and the maximum magnitude of the morning/evening amplitude of the sun, making observations at times from the doorstep of the entrance (and a little to the left), and at times from the base of the area under the dome, watching either inner or outer edges of the windows. Neither of this has sense, because to whom, and what for, would the master builder leave such a "record"? The obliquity of the ecliptic to the equator can be indirectly recorded along the meridian line by making marks on the ends of the solstice shadow, while materializing the maximum morning/evening amplitude of the sun makes sense only if the main axis of the church is oriented according to the rule, along the West-East direction (Figure 7, left).

10. Horizontal and vertical church plan is in accordance with the noon amplitudes of the sun during both summer and winter solstice and both spring and autumn equinox (pg. 70, Figure 32).

Comment. In construction, providing enough sunlight for the interior space is an important task, both from the point of view of human health and aesthetics; optimal sunlight is achieved by adapting the orientation, the shape and the position of the building, as well as the arrangement and size of the windows. Planning is based on knowing the geometry of the Sun for a given standing point during solstice and equinox, and especially the noon amplitudes of the Sun, which are, figuratively speaking, materialized in the building itself. We cannot say this for the church of the Holy Cross; the sketches which supposedly prove this actually mislead the observer into believing that the main axis of the church coincides with the meridian line (Figure 29–32).

11. Each opening on the church wall has been positioned and deformed intentionally, in order to direct the sun rays during solstices and equinoxes to specific spots in order to inform the observers that the moment has come to start one of the daily prayers (according to the temporal hour system) (pg. 72–86, Figure 36–45).

Comment. At the time of one given prayer (of the same temporal hour), e.g. tertia, (mid morning), or nona (mid afternoon), the sun is not positioned along the same direction, but rather changes its azimuth within a wide range, which at the geographic latitude of the church of the Holy Cross measures 54° 40' (Figure 7, right), so that a beam of sunlight, coming through one of the windows on the wall, describes a corresponding hyperbolic line on the church floor. That's why the positions of reflections from the windows on the floor and on the walls of the church during the year are impossible to memorize, and to mark them only during solstices and equinoxes is quite useless. Figures 36-38 are created in such a way that they do not prove the statement, but rather bring it under suspicion, owing to a multitude of obviously excessive details.

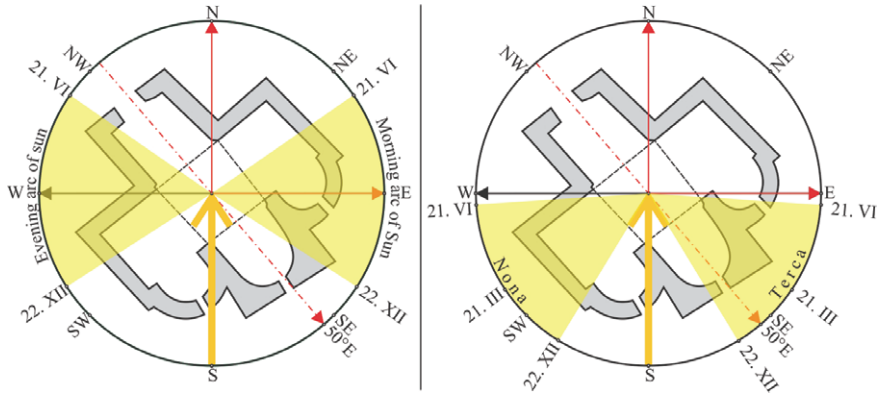


Figure 7. Oriented ground plan of the church of the Holy Cross with the prominent horizon sectors of the rising and setting sun (left), and sectors within which moves the Sun at the time of a tertia and nona (right)

12. A direct check in 1976 (21st June, 22nd September) and 1977 (21nd December) has confirmed the correctness of the predictions (pg. 139–160, Figure 38–76).
Comment. The attached photographs, without the given time when they were taken (h:min), do not prove that photographed effects are a result of intentional church modelling, and not incidental external effects of the shadows that the walls cast one upon the other, or effects created in the interior by reflected of sunbeams coming through the church windows. Not to mention that this checking, owing to the differences between the Julian and the Gregorian calendar, should have taken place not on 21st June and 22nd December, but six days later; ten days prior and ten days after the solstice the sun rises practically at one same point (it moves only 1°), so that morning pictures of the church walls at this time do not provide us with any trustworthy proves.

Conclusion

Geographical azimuth of the main axis of the church of the Holy Cross is approx. $A_N = 140°$, which means that it is positioned outside the horizon sector of the rising sun, i.e., that the church should be oriented "to the East" or "to sunrise" (Mirković, 1966). Such orientation of the church of the Holy Cross is dictated by the position of a group of older buildings among which the church is constructed, the buildings whose foundations (positioned at the right angle to the direction of the strong winter wind blowing in the region, called "bura") are clearly visible on the orthophoto map of the city of Nin.

The designed project ground plan of the church is an even-armed cross, a Greek cross. After flattening the terrain, the medieval master builder has first constructed a square and then drawn a cross inside it. In the ideal case, the square would be oriented along the cardinal directions, and in the case of the church of the Holy Cross it has been rotated by 50° from East to Southeast. That is why the diagonals of the imaginary square that describes the ground plan of the church of the Holy Cross deviate by 5° from the cardinal directions. So, this angle has no hidden meaning.

The medieval master builder had very simple tools, a rope with knots and a try square, so it is no wander that he was not able to construct the ground plan precisely. He did not obtain right angles, he distorted the central square of the area under the dome and the arms of the Greek cross, he tilted the walls and deformed the semi-circular apses, the tambour and the dome, and continued in the similar fashion, creating an asymmetrical arrangement of windows and niches. Such deviations were quite common for a majority of churches of that period, the master builders were simply incapable of carrying out a construction according to the project: "Everywhere [in old Croatian buildings] can we find deviations; a circle is more an ellipse than a circle, the squares are unevenly sided, with irregular angles, the arrangement of windows and niches can hardly show any symmetry. We can also find irregularities in the cross sections of the buildings, the window arches are often dislocated from the axes, the inner and outer silhouettes of the openings are uneven. These small churches are actually a collection of irregularities which we can observe in all parts" (Dobrić, n. d.).

That is how the church of the Holy Cross has been created, as a set of different and easily visible constructional irregularities. From the interpretation that painter M. Pejaković has broadly discussed in his book, we can conclude that the master builder's intention was not to exactly fulfil the architectural project. Instead, he intentionally distorted it, with the intention of transforming the

church into an astronomic calendar and prayer clock, so it can be concluded that the master builder, except for the proficient knowledge of the calendar, knew how to determine the cardinal points, was familiar with the altitude of the north celestial pole (climat) and morning and evening amplitudes of the sun, and knew how to calculate exact time according to the temporal hour system and horizontal coordinates of the sun at the moments of daily prayers. This further means that the master builder had an excellent knowledge of "De architectura Libri X" by Vitruvius, and was in possession of an enviable amount of knowledge in astronomy, i.e., that he was a top architect of the age. And had he been a top architect, the church of the Holy Cross would now look completely different.

Trying to prove his hypothesis, M. Pejaković "adjusted" the facts to fit it, constructed numerous, often misleading sketches, "stretched" angles and shifted the observation point (centre of the area under the dome, centre of the basis of the area under the dome, the point on the entrance door), giving us very complicated explanations, which themselves arise the reader's suspicion. If all his statements were true, was there no simpler way to explain them? Having succumbed to his own fantasies, he created a fiction which, obviously, none of the competent experts wanted to question, maybe fearing that this could be misinterpreted as an attack upon the entire Croatian national architecture. This kind of story could be created around the relation of light and shadow of any church, whatever its orientation, especially if the church has an elaborate ground plan. During each of the 365 days of the year, of which every single one is dedicated to a saint or an event from the church calendar, every day the Sun follows a new apparent path above the horizon, rising and setting at the new points of the physical horizon and achieving different meridian amplitudes, causing the church walls to cast, receive and refract different shadows, while, through the windows, beams of sunlight penetrate the church interior from different directions and under different angles, falling onto different spots on the church floor and church walls, and also on the frescoes, and the icons of painted churches, which is just about a perfect setting for fiction.

Overwhelmed by the hypothesis he created about the solar meaning of the church of the Holy Cross, M. Pejaković has constructed his own card castle, but by this he drew attention to the importance of conveying exact and careful investigations of orientation of medieval churches, and this is the principal reason for which he should be given credit. As for the church of the Holy Cross, it really doesn't need any fake "solar apparel", it is quite beautiful, in all its rustically simplicity, without it.

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